Find the power series for $f(x)=\dfrac{x}{(1+4x)^2}$, and the associated **radius** of convergence. **SCORE: _____ / 6 PTS**

$$\frac{1}{(1+4x)^2} \overset{1}{=} -\frac{1}{4}\frac{d}{dx}\frac{1}{1+4x} = -\frac{1}{4}\frac{d}{dx}\frac{1}{1-(-4x)} = -\frac{1}{4}\frac{d}{dx}\sum_{n=0}^{\infty}(-4x)^n = -\frac{1}{4}\frac{d}{dx}\sum_{n=0}^{\infty}(-4)^n x^n \quad 1$$

$$= -\frac{1}{4}\sum_{n=0}^{\infty}\frac{d}{dx}(-4)^n x^n = -\frac{1}{4}\sum_{n=0}^{\infty}(-4)^n n x^{n-1} \overset{1}{=} \sum_{n=1}^{\infty}(-4)^{n-1} n x^{n-1}$$

$$\frac{x}{(1+4x)^2} = x\sum_{n=1}^{\infty}(-4)^{n-1} n x^{n-1} = \sum_{n=1}^{\infty}(-4)^{n-1} n x^n \quad 1$$

differentiation does not affect radius of convergence, so $|-4x|<1 \Rightarrow -1<-4x<1 \Rightarrow -\frac{1}{4}<x<\frac{1}{4}$ 1

radius of convergence $=\frac{1}{4}$ 1

Find the first 4 non-zero terms of the Taylor series for $f(x) = \tan x$ centered at $x = \frac{\pi}{4}$. SCORE: _____ / 7 PTS

$f(x) = \tan x$ $\qquad$ $f(\frac{\pi}{4}) = 1$

$\frac{1}{2}$ $f'(x) = \sec^2 x$ $\qquad$ $f'(\frac{\pi}{4}) = (\sqrt{2})^2 = 2$

1 $f''(x) = 2\sec^2 x \tan x$ $\qquad$ $f''(\frac{\pi}{4}) = 2(\sqrt{2})^2(1) = 4$

1 $f'''(x) = 4\sec^2 x \tan^2 x + 2\sec^4 x$ $\qquad$ $f'''(\frac{\pi}{4}) = 4(\sqrt{2})^2(1)^2 + 2(\sqrt{2})^4 = 16$

$$\tan x = \frac{1}{0!} + \frac{2}{1!}(x - \tfrac{\pi}{4}) + \frac{4}{2!}(x - \tfrac{\pi}{4})^2 + \frac{16}{3!}(x - \tfrac{\pi}{4})^2 + \dots = \underbrace{1}_{\frac{1}{2}} + \underbrace{2(x - \tfrac{\pi}{4})}_{1} + \underbrace{2(x - \tfrac{\pi}{4})^2}_{1\frac{1}{2}} + \underbrace{\tfrac{8}{3}(x - \tfrac{\pi}{4})^2}_{1\frac{1}{2}} + \dots$$

Find the first 4 non-zero terms of the Maclaurin series for $f(x) = \sqrt[3]{1+x^2}$ **without using differentiation**. SCORE: _____ / 4 PTS

$$(1+x^2)^{\frac{1}{3}} = 1 + \tfrac{1}{3}(x^2) + \frac{\frac{1}{3}(-\frac{2}{3})}{2!}(x^2)^2 + \frac{\frac{1}{3}(-\frac{2}{3})(-\frac{5}{3})}{3!}(x^2)^3 + \ldots$$

$$= 1 + \tfrac{1}{3}x^2 - \tfrac{1}{9}x^4 + \tfrac{5}{81}x^6 + \ldots$$

$\frac{1}{2}$ $\frac{1}{2}$ $1\frac{1}{2}$ $1\frac{1}{2}$

Find the power series for $f(x) = \dfrac{3}{x^2 - x - 2}$ by first using partial fractions decomposition. **SCORE: _____ / 6 PTS**

$$\frac{3}{x^2 - x - 2} = \frac{3}{(x-2)(x+1)} = \frac{A}{x-2} + \frac{B}{x+1} = \frac{A(x+1) + B(x-2)}{(x-2)(x+1)} \Rightarrow A(x+1) + B(x-2) = 3$$

1

Let $x = -1 \Rightarrow B(-3) = 3 \Rightarrow B = -1$

Let $x = 2 \Rightarrow A(3) = 3 \Rightarrow A = 1$

1 1

$$\frac{1}{x-2} - \frac{1}{x+1} = -\frac{1}{2}\frac{1}{1-\frac{x}{2}} - \frac{1}{1-(-x)}$$

$$= -\frac{1}{2}\sum_{n=0}^{\infty}\left(\tfrac{x}{2}\right)^n - \sum_{n=0}^{\infty}(-x)^n = \sum_{n=0}^{\infty} -\tfrac{1}{2^{n+1}}x^n - \sum_{n=0}^{\infty}(-1)^n x^n = \sum_{n=0}^{\infty}\left[(-1)^{n+1} - \tfrac{1}{2^{n+1}}\right]x^n$$

1 1 1

Find the **interval** of convergence of the series $\displaystyle\sum_{n=1}^{\infty}\frac{3^n(x+2)^n}{\sqrt{n}}$.

SCORE: _____ / 7 PTS

ALL ITEMS WORTH $\frac{1}{2}$ POINT UNLESS OTHERWISE NOTED

centered at $x=-2$

radius of convergence $= \displaystyle\lim_{n\to\infty}\left|\frac{3^n}{\sqrt{n}}\frac{\sqrt{n+1}}{3^{n+1}}\right| = \lim_{n\to\infty}\left|\tfrac{1}{3}\sqrt{1+\tfrac{1}{n}}\right| = \tfrac{1}{3}\sqrt{1+0} = \tfrac{1}{3}$

at $x=-2+\frac{1}{3}=-\frac{5}{3}$, $\displaystyle\sum_{n=1}^{\infty}\frac{3^n(x+2)^n}{\sqrt{n}} = \sum_{n=1}^{\infty}\frac{3^n(\frac{1}{3})^n}{\sqrt{n}} = \sum_{n=1}^{\infty}\frac{1}{\sqrt{n}}$ diverges ($p=\frac{1}{2}<1$) by p-Series Test 1

at $x=-2-\frac{1}{3}=-\frac{7}{3}$, $\displaystyle\sum_{n=1}^{\infty}\frac{3^n(x+2)^n}{\sqrt{n}} = \sum_{n=1}^{\infty}\frac{3^n(-\frac{1}{3})^n}{\sqrt{n}} = \sum_{n=1}^{\infty}\frac{(-1)^n}{\sqrt{n}}$ converges by Alternating Series Test 1

since $\dfrac{1}{\sqrt{n}}\to 0$ and is decreasing

interval of convergence $=[-\frac{7}{3},-\frac{5}{3})$

CORRECT DELIMITERS [,) OR ≤ <

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